Formal Semantics: Disjunctions

Disjunctions are the last of the molecular sentences, the product of the fourth construction rule.

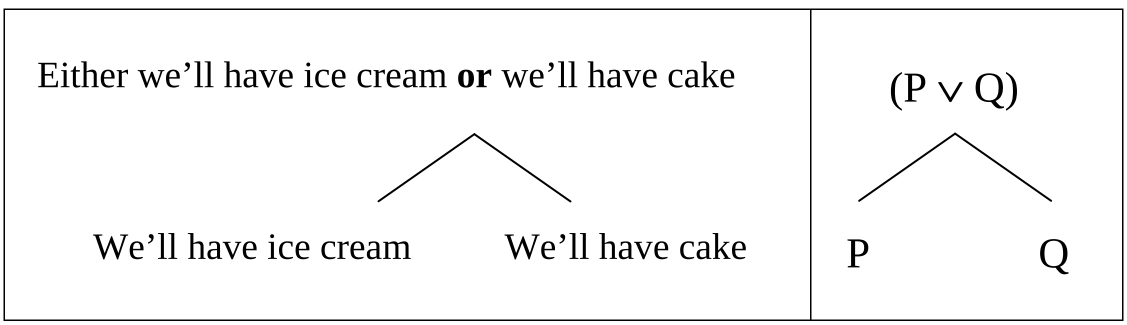
> 4. If ● and ▲ are formal sentences, then (● $\vee$ ▲) is a formal sentence.

As always, we take a small English sentence as our guide.

> P: We'll have ice cream
> Q: We'll have cake.

> Either we'll have ice cream, or we'll have cake (P $\vee$ Q)

The construction tree is similar to the tree for a conjunction.



The truth table will follow the construction, in its horizontal way.

P	**Q**	**(P $\vee$ Q)**

With two sentence letters, the truth table calls for four valuations.

4 = 2 x 2

P	**Q**	**(P $\vee$ Q)**
1	1	
1	0	
0	1	
0	0	

The first valuation resents a situation where it's true that we'll have ice cream, and true that we'll have cake. Now recall that the vel is intended as an **inclusive** disjunction: "(P ∨ Q)" means "P, or Q, or possibly both". With that in mind, we see that the first valuation, where we have both ice cream and cake, makes the disjunction **true**.

	P	**Q**	**(P ∨ Q)**
⇨	1	1	**1**
	1	0	
	0	1	
	0	0	

In the second valuation we have ice cream, but no cake. While the disjunction allows for the possibility of both, it only *promises* that we'll have one or the other. In an ice-cream-enhanced-but-cake-free situation such as Valuation 2, that promise is kept. So long as we have at least one, the disjunction is **true**.

	P	**Q**	**(P ∨ Q)**
	1	1	1
⇨	1	0	**1**
	0	1	
	0	0	

For the same reason, in the third valuation – where we have no cake, but ice cream –the disjunction is also true.

	P	**Q**	**(P ∨ Q)**
	1	1	1
	1	0	1
⇨	0	1	**1**
	0	0	

In the fourth valuation it's false that we'll have ice cream, and false that we'll have cake. Here the disjunction is **false**: a prediction of ice cream or cake, followed by neither, is a false prediction.

P	**Q**	**(P $\vee$ Q)**
1	1	1
1	0	1
0	1	1
0	0	**0**

The same pattern holds for *any* inclusive disjunction, regardless of subject matter. So we present the semantic rule for disjunctions in full generality.

Disjunction Rule

●	▲	(● $\vee$ ▲)
1	1	1
1	0	1
0	1	1
0	0	0

Compared to conjunctions, a disjunction is quite easy to please: a disjunction is true as long as at least one of its parts is true. Put the other way around: **a disjunction is only false when both its parts are false**.

We find a striking symmetry between conjunctions and disjunctions, where truth and falsehood are concerned.

	conjunction		**true**		**true**
A		is only		when both its parts are	
	disjunction		**false**		**false**

We will later explore this symmetry in considerable detail. But already it makes for a simple way of remembering the semantic rules for conjunctions and disjunctions.